Focusing of an Intense Relativistic Electron Beam by a Hollow Conical Laser Beam *

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Estimates suggest that the nonlinear transverse radiation pressure produced within a plasma by a convergent annular high power laser beam may lead to the focusing of an intense relativistic electron down to a radius of $\sim 10^{-4}$ cm. The transverse radiation pressure results from the dielectric property of a plasma in conjunction with the phenomena of the self-focusing of intense laser light. The tightly focused electron beams would make possible the release of thermonuclear energy by micro-explosions.

In concepts of controlled thermonuclear energy release by micro-explosions initiated with intense relativistic electron beams one of the principal problems is the focusing of these beams onto a very small area. It had been shown [1] that the focusing into dense matter of an intense relativistic electron beam with beam currents in excess of $\sim 10^6$ Ampere, down to a diameter of $\sim 10^{-3}$ cm promises the release of thermonuclear energy by microexplosions with remarkably small beam energy inputs. The beam would then form a very narrow pinch channel with a very strong magnetic field of the order $\sim 10^8$ Gauss, which by action of magnetic confinement would

a) delay the thermal expansion of the narrow pinch channel,

b) reduce the electronic heat conduction losses to the surrounding dense plasma in the radial direction, and

c) confine the charged fusion products to within the pinch channel and hence reacting region which is the condition for a thermonuclear detonation.

Although the question of stability for such a tightly confined electron beam within dense material is still unresolved, there are at least three reasons favoring a higher degree of stability than in an ordinary pinch discharge: 1) The pinch would be enclosed by a plasma with solid state density acting against hydrodynamic instabilities [2]. 2) The fact that the current would be carried by relativistic electrons higher stability is expected due to radiation damping. 3) Since the beam propagating freely in dense matter can only establish a high magnetic field after repelling, its return current by the Weibel instability, a process which is very fast and much shorter than the beam discharge time, the high current filament will be surrounded by an annular return current sheet which is likely to enhance magnetohydrodynamic stability.

In fact, with intense relativistic electron beams, stable pinches for the time scale of the beam discharge time of $\sim 10^{-8}$ sec and with a diameter of $\sim 10^{-1}$ cm have been observed within the high voltage diode by the attachment of a small guide electrode onto the center of the cathode surface facing the anode [3, 4]. However, attempts by this methode to focus the beam down to a smaller diameter have so far been unsuccessful or at least inconclusive. We will give here the likely reason for this failure and propose a way for focusing the beam down to substantially smaller diameters.

A beam current I (Ampere) focused down to a radius r (cm) will have a selfmagnetic field $H = 0.2\, I/r$ with a magnetic stress $H^2/4\pi = (10^{-2}/\pi)(I/r)^2$. Since the guide electrodes to focus the beam have a finite tensile strength σ it is obvious that in order to prevent their mechanical destruction by the magnetic forces of the beam one must require that $H^2/4\pi < \sigma$. A typical value for the tensile strength of a solid is $\sigma \sim 10^{10}$ dyn/cm² resulting in $H \lesssim 3 \times 10^5$ Gauss or $I/r \lesssim 10 \sqrt{\pi \sigma} \cong 1.8 \times 10^6$ Ampere/cm. This shows that the concentration of a current in the megampere range by a solid guide electrode down to a diameter much less than ~ 1 cm, as it is contemplated for thermonuclear micro-explosions, will be difficult to achieve.

In order to overcome these limitations we propose here the following technique explained in Fig. 1

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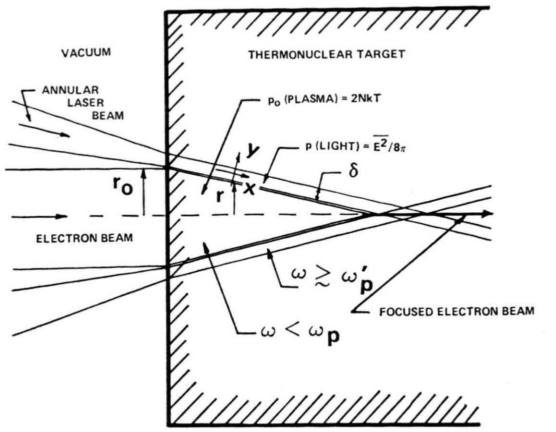

Fig. 1. The focusing of an intense relativistic electron beam by a conical annular laser beam.

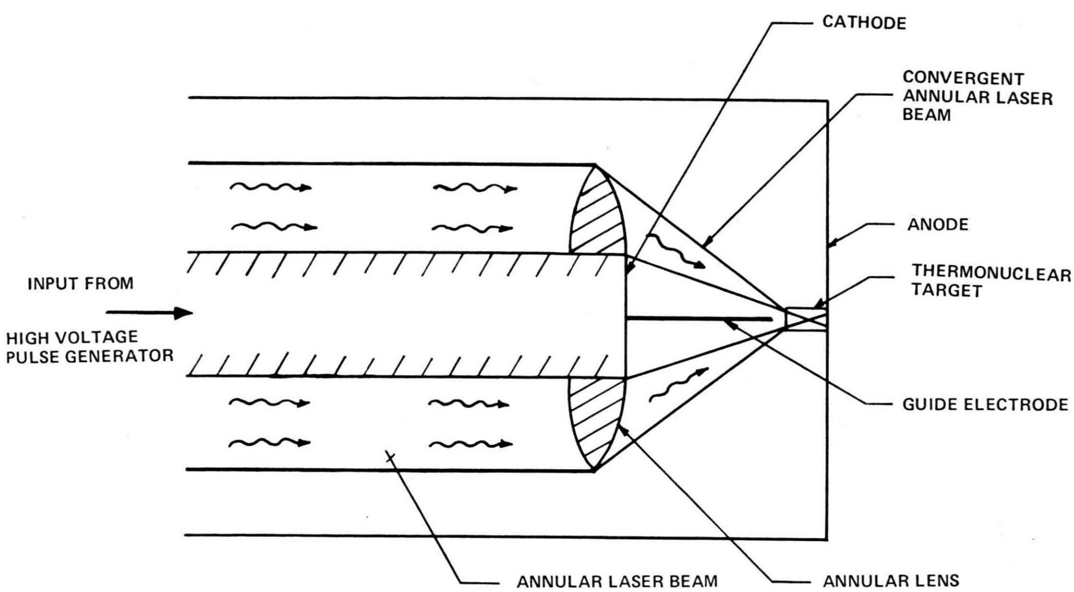

Fig. 2. The bombardment and focusing of the thermonuclear target within the diode space by the electron beam with the annular laser beam.

and Figure 2. Figure 1 shows an axial cross-section of a relativistic electron beam of initial radius r_0 projected into a thermonuclear target. As seen in Fig. 2 the beam is first discharged into the target by a guide electrode. But also any other method producing an initially well collimated beam would serve the same purpose. The target is grounded to the anode so that the beam is focused within the diode space. The target itself may be prepared in form of a solid piece of thermonuclear material, for example TD, which upon interaction with the beam is transformed into a plasma. A short time before the beam is projected onto the target an annular conical convergent high intensity pulsed laser beam is shot into the target with a cone radius $r \gtrsim r_0$ at impact as shown in Fig. 1 and Figure 2. In Fig. 2 one can see how this can be done by an annular laser beam, which after propagating along the cathode is focused onto the target by an annular lens positioned around the end of the cathode. The laser frequency ω shall be chosen such that $\omega < \omega_p$ where ω_p is the plasma frequency of the target material. This condition is met by a high intensity IR laser as for example a CO_2 or a HF chemical laser. The annular laser beam projected onto the thermonuclear targed shall have at its impact a width which is close the diffraction limit and at the high contemplated intensities by action of optical self-focusing will form a narrow convergent annular plasma channel of width δ, which by thermal expansion will then form an annular plasma region with plasma frequency ω_p' for which $\omega_p' \lesssim \omega < \omega_p$ making it optically transparent and permitting the laser pulse to propagate within this channel. We will henceforth distinguish in between two limiting cases. In the first case we will assume that after the channel has become optically transparent no further appreciable heating and hence thermal expansion will take place such that from there on ω_p' will remain just slightly smaller than ω. In the second case we will assume that the heating leads to a substantial plasma expansion such that $\omega_p' \ll \omega$. These two limiting cases will give an upper and lower estimate for the pondermotive force acting on the plasma. When a laser beam with $\omega < \omega_p$ is focused onto a solid targed a crater will form in which the average plasma density is considerably lower than the critical one, that is one will have $\omega_p' \ll \omega$ [5]. However, one may also form the annular channel by a short wave length laser prepulse with $\omega_0 \gtrsim \omega_p$ prior to the main pulse propagating through the thusly formed channel region in which then due to thermal expansion $\omega \gtrsim \omega_p'$. But in this case the formed plasma with $\omega_p' \cong \omega$ exhibits parametric instabilities by which it is difficult to avoid $\omega_p' > \omega$ at some point leading to light reflection [6, 7]. At the other hand the phenomena of optical selffocusing for the laser light is in support for a plasma state with $\omega_p' \lesssim \omega$ rather than $\omega_p' \ll \omega$. The time to form the plasma channel is of the order l/v_s where l is the length of the channel and v_s the velocity of the shock wave generated by the laser pulse propagating into the target. For $l \cong 1$ cm and $v_s \cong 10^7$ cm/sec this time would be $\sim 10^{-7}$ sec.

The laser radiation confined within the annular convergent channel will create a transverse radiation pressure which can be easily computed from Maxwell's stress tensor in a dielectric medium [8, 9], with the total force density acting on the plasma given by

$$\boldsymbol{f} = \nabla \cdot \left[\sigma_{ik} + \frac{\varepsilon - 1}{4\pi} E_i E_k - p\,\delta_{ik}\right] + \frac{\varepsilon - 1}{4\pi c} \frac{\partial}{\partial t} \boldsymbol{E} \times \boldsymbol{H}, \quad (1)$$

where σ_{ik} is the Maxwell stress tensor in vacuum and p the total pressure within the plasma. The dielectric property of the plasma is given by

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\left[1 + i\,\frac{\nu}{\omega}\right], \quad (2)$$

where ν is the electron collision frequency. Since the radiation pressure shall be computed in the plasma channel where $\omega_p = \omega_p'$ one has to substitute into Eq. (2) ω_p' for ω_p. The two limiting cases, the one for which ω_p' is just slightly smaller than ω is then distinguished by $\mathrm{Re}\,\sqrt{\varepsilon} =$ minimum and the other for which $\omega_p' \ll \omega$ by $\varepsilon = 1$. If the propagating of the annular light beam is along an x-axis and the direction perpendicular to it along a y-axis of a local cartesian coordinate system (see Fig. 1), then we can distinguish between two polarizations for which $E_x = E_z = H_x = H_y = 0$ and $E_x = E_y = H_x = H_z = 0$. For an electromagnetic wave propagating in a dielectric substance one has for the time average $\overline{H^2} = \varepsilon\,\overline{E^2}$ and hence for both the first and second polarization case

$$f_y = -\frac{\partial}{\partial y}\left[\frac{1-\varepsilon}{8\pi}\,\overline{E^2} + p\right], \quad (3)$$

where $\overline{E_y^2} = \overline{E_z^2} \equiv \overline{E^2}$. The equilibrium condition is given by $f_y = 0$, hence

$$(1 - \varepsilon)\,\overline{E^2}/8\pi + p = \text{const.} \quad (4)$$

The square average of the electric field $\overline{E^2}$ in the plasma is related to the square average of the vacuum electric field of the incident laser radiation by $\overline{E^2} = \overline{E_0^2}/\sqrt{\varepsilon}$, hence it follows from Eq. (4)

$$\frac{1-\varepsilon}{\sqrt{\varepsilon}}\,\frac{\overline{E_0^2}}{8\pi} + p = \text{const.} \quad (5)$$

In the dense plasma region where $\omega < \omega_p$ the laser light intensity decreases rapidly over the distance $\sim c/\omega_p$, therefore $\overline{E^2} \cong 0$ and $p = p_0$ where p_0 is the maximum hydrostatic pressure within the plasma, therefore const $= p_0$ and thus

$$\frac{1-\varepsilon}{\sqrt{\varepsilon}}\,\frac{\overline{E_0^2}}{8\pi} + p = p_0\,. \quad (6)$$

For the upper limiting case with $\omega \gtrsim \omega_p'$ we proceed as follows. For $\nu = 0$ and $\omega = \omega_p'$ one would have [from Eq. (2) putting $\omega_p = \omega_p'$] $\varepsilon = 0$, and $\overline{E^2} = \infty$. The radiation pressure would therefore diverge. However, if ν is finite $\overline{E^2}$ remains finite. At the contemplated laser intensities the collision frequency is not determined by the classical electron-ion collision frequency which normally is very small compared to ω_p and which would lead to very large radiation pressures but rather by the growth rate of the oscillating two-stream instability [10], and which approximately is given by $0.7\,\omega_{pi} = 0.7\ (m/M)^{1/2}\,\omega_p \cong \nu$, where ω_{pi} is the ion plasma frequency and m/M the electron ion mass ratio [11]. Recognizing that $\nu/\omega_p \ll 1$ one obtains from Eq. (2) the approximation

$$\varepsilon \cong 1 - \frac{\omega_p^2}{\omega^2}\left[1 + 0.7\,i\left[\frac{m}{M}\right]^{1/2}\frac{\omega_p}{\omega}\right]. \quad (7)$$

For $\omega \cong \omega_p'$ [putting in Eq. (7) $\omega_p = \omega_p'$] one would then have with sufficient accuracy $1 - \varepsilon \cong 1$ and $\mathrm{Re}\,|\sqrt{\varepsilon}| = 0.6\ (m/M)^{1/4}$, hence $(1 - \varepsilon)/\sqrt{\varepsilon} \cong 1.7$

$(M/m)^{1/4}$. For a TD plasma this factor would be $\cong 14$. If the pressure in the laser channel is small as compared to the pressure in the dense plasma one obtains for the equilibrium condition

$$p_0 = 1.7 \left[\frac{M}{m}\right]^{1/4} \frac{\overline{E_0^2}}{8\pi} \simeq \frac{\overline{E^2}}{8\pi}. \tag{8}$$

At the other extreme where $\varepsilon = 1$ one has $p_0 = \overline{E_0^2}/8\pi$. With the average Poynting vector $S = c\overline{\mathbf{E} \times \mathbf{H}}/4\pi = c\overline{E_0^2}/4\pi$ one can therefore write for the lower and upper estimate

$$S/2c < p_0 < 1.7\,(M/m)^{1/4}\, S/2c, \tag{9}$$

or in expressing S by the total laser light power P and the area A onto which the light is focused one has $S = P/A$ and

$$P/2Ac < p_0 < 1.7\,(M/m)^{1/4}\, P/2Ac. \tag{10}$$

The pressure p_0 in the plasma is composed of essentially three parts 1) the pressure of the plasma ions and electrons $2NkT$, 2) the pressure of the relativistic electrons in the beam and 3) the magnetic pressure of the selfmagnetic field generated by the current of the intense electron beam. For a self-confined beam the forces resulting from the pressure of the beam electrons and beam magnetic field are in equilibrium, hence $p_0 = 2NkT$. In order to obtain beam pinching, the beam magnetic field has to be trapped within the target plasma, which will happen if the target was initially in a cold low conducting state prior to the transformation of the target by beam heating into a highly conducting plasma [1]. The beam magnetic field and hence the beam itself are thereafter confined within the thusly formed plasma, and if the electron beam propagates into the plasma confined by the convergent annular light beam, it too will be confined to the plasma and hence focused down to a small diameter as it approaches the vertex point of the conical light beam. The maximum beam focusing is determined by the width δ of the self-focused laser radiation. Experimentally values as small as $\delta \sim 10^{-4}$ cm have been observed. The maximum beam focusing is determined by the confinement condition Eq. (10) which now reads

$$1.7\,(M/m)^{1/4}\, P/2Ac > 2NkT > P/2Ac. \tag{11}$$

For a plasma radius and hence beam radius r somewhere within the converging annular light beam channel of width δ one has $A \cong 2\pi r \delta$. For a target material with $N = 5 \times 10^{22}\ \text{cm}^{-3}$ (i.e. solid TD) and expressing kT in eV one has

$$6 \times 10^{22}\, \text{T} r \delta > P > 4.3 \times 10^{21}\, \text{T} r \delta. \tag{12}$$

The plasma temperature in the converging channel is likely to increase as the beam approaches its highest concentration. The most important mechanism to heat the plasma is by the two-stream instability. However, if sufficient collisions take place in between the beam electrons and the background plasma, the growth of the two-stream instability can be suppressed. This could be done by increasing the scattering cross section with the addition of high-Z impurities in that part of the plasma where the focusing takes place. The admixture of high-Z material has the additional advantage of slowing down the growth rate of the oscillating two-stream instability thus making the effective collision frequency ν entering Eq. (2) smaller and hence $\overline{E^2}$ larger leading to an increased transverse radiation pressure $\overline{E^2}/8\pi = \overline{E_0^2}/8\pi\sqrt{\varepsilon}$. In case of the electron beam what remains is target heating by classical stopping power, a process which is not very efficient. It therefore, may be possible to keep the plasma temperature in the focusing region below ~ 100 eV ($\sim 10^6$ °K).

Assuming beam focusing down to $r \sim 10^{-3}$ cm with $\delta \sim 10^{-4}$ cm would require 6×10^{17} erg/sec $> P > 4.3 \times 10^{16}$ erg/sec, which could be accomplished by a laser pulse energy e, of 600 J $> e >$ 43 J to be delivered in $\sim 10^{-8}$ sec and which would correspond to a plasma and radiation pressure of 1.6×10^{13} dyn/cm$^2 \cong 16$ Mb.

In order to keep the focused electron beam stable against hydrodynamic instabilities one has to require that

$$\frac{\overline{E_0^2}}{8\pi} \le \frac{\overline{H^2}}{8\pi} \le \frac{\overline{E^2}}{8\pi} \le 14\, \frac{\overline{E_0^2}}{8\pi}, \tag{13}$$

where $H = 0.2\, I/r$ is the beam magnetic field. Condition (13) therefore reads

$$\frac{P}{2Ac} < \frac{10^{-2}}{2} \left[\frac{I}{r}\right]^2 < 14\, \frac{P}{2Ac}. \tag{14}$$

For $r = 10^{-3}$ cm, $I = 2 \times 10^6$ Ampere and $\delta = 10^{-4}$ cm one obtains 2.4×10^{20} erg/sec $> P > 1.7 \times 10^{19}$ erg/sec which could be achieved with 2.4×10^5 J $> e > 1.7 \times 10^4$ J laser pulse in 10^{-8} sec. In this case the radiation pressure would be $\sim 6.4 \times 10^{15}$ dyn/cm^2 = 6400 Mb.

After the beam has been focused down it may be projected into dense plasma placed just behind the anode and may from there on propagate as a freely drifting current neutralized beam. In this case then, because of the fast growing Weibel instability, the return current in the dense plasma may be expelled from the beam resulting in localized very strong magnetic fields as they are desirable for electron beam induced thermonuclear reactions. It is therefore suggested that with the proposed method thermonuclear reactions not only can be initiated in the diode space but also in the drift space behind the diode filled with dense plasma.

In conclusion it can be stated that there still remain many unanswered questions which will decide the feasibility of the proposed beam focusing method. As an example we may cite the question for the lifetime of the conical channel filled with laser light. One may think that the channel, due to thermal expansion, has a very short lifetime of the order δ/v_s. This however, is probably not the case, since the thermal expansion is compensated in part by the energy supply from the laser beam replenishing the channel with radiation, which in effect will lead to the development of two conical shockwaves, one converging onto the region confining the electron beam and one diverging from it, thereby communicating the radiation pressure to the solid material which ultimately is responsible by action of its inertia for the confinement process. In spite of this reasoning the thermal expansion of the plasma channel is the greatest uncertainty and requires further study.

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